

# QuaternaryProd

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A signed causal graph for gene regulation is a directed graph where the edges are signed and the signs indicate the direction of regulation of the target genes (the signs are either (+) or (-)). **QuaternaryProd** is a package for computing the Quaternary Dot Product Scoring Statistic [2] (or simply the Quaternary Statistic) of signed causal graphs for gene regulation. The Quaternary Dot Product Scoring Statistic is a generalization of the Ternary Dot Product Scoring Statistic [1] which allows for ambiguities to arise in the causal graph. Ambiguities arise when a regulator can affect a target gene in two different ways or if the direction of regulation is unknown. We will first provide some background, and then we will apply the statistic to STRINGdb [3] which is a publicly available biological network.

## Introduction

The Quaternary Dot Product Scoring Statistic [2] is a goodness of fit test for evaluating the performance of regulation predictions made by a signed and directed causal network on a given gene expression data set. Given a regulator  $s$  in a causal graph, let  $q_p$ ,  $q_m$  and  $q_r$  denote the number of target genes which are upregulated (+), downregulated (-) and regulated (r) by the regulator  $s$  respectively. Regulated relations occur when a regulator regulates a target gene without knowing the direction of regulation or if an ambiguity in direction of regulation occurs. An ambiguity can occur if a regulator, according to a given network, shares both (+) and (-) relations with the same target gene. Moreover, let  $q_z$  denote the set of target genes in the causal network which do not share a relation with  $s$  i.e which are not affected by  $s$ . Next, suppose we are presented with new gene expression data. Let  $n_p$ ,  $n_m$  and  $n_z$  denote the number of genes which are upregulated, downregulated and are unregulated in the gene expression data respectively. For the regulator  $s$ , we can tabulate the predictions of the network vs. the gene expression data:

	Observed +	Observed -	Observed 0	Total
Predicted +	$n_{pp}$	$n_{pm}$	$n_{pz}$	$q_p$
Predicted -	$n_{mp}$	$n_{mm}$	$n_{mz}$	$q_m$
Predicted $r$	$n_{rp}$	$n_{rm}$	$n_{rz}$	$q_r$
Predicted 0	$n_{zp}$	$n_{zm}$	$n_{zz}$	$q_z$
Total	$n_p$	$n_m$	$n_z$	$T$

Table 1: Tabulation of predictions from network edges vs. observations from experimental results.

$n_{pp}$  denotes the number of target genes which  $s$  is predicted to upregulate by the network and were indeed upregulated in the gene expression data.  $n_{pm}$  denotes the number of target genes which  $s$  is predicted to upregulate and were downregulated in the gene expression data.  $n_{pz}$  denotes the number of target genes which  $s$  is predicted to upregulate and were not expressed in the gene expression data. Similar interpretation follows for all other entries of the table. The probability of a tabulation table follows the Quaternary Dot Product distribution which is given by:

$$P(\text{Table}) = \frac{\binom{q_p}{n_{pp}, n_{pm}, n_{pz}} \binom{q_m}{n_{mp}, n_{mm}, n_{mz}} \binom{q_z}{n_{zp}, n_{zm}, n_{zz}} \binom{q_r}{n_{rp}, n_{rm}, n_{rz}}}{\binom{T}{n_p, n_m, n_z}}. \quad (1)$$

Note, since the predictions by the network and the experimental values are fixed, then the table has 6 degrees of freedom  $n_{pp}$ ,  $n_{mm}$ ,  $n_{rp}$ ,  $n_{rm}$ ,  $n_{mp}$  and  $n_{pm}$ . The score  $S$  to measure the goodness of fit is given by:

$$S(\text{Table}) = n_{pp} + n_{mm} + n_{rp} + n_{rm} - (n_{mp} + n_{pm}) \quad (2)$$

which is the sum of the good predictions (i.e  $n_{pp}$ ,  $n_{mm}$ ,  $n_{rp}$  and  $n_{rm}$ ) minus the bad predictions (i.e  $n_{mp}$  and  $n_{pm}$ ). To compute the probability of a score, we sum the probabilities of all tables with score  $S$  as follows:

$$P(S) = \sum_{P(\text{Table})=S} P(\text{Table}). \quad (3)$$

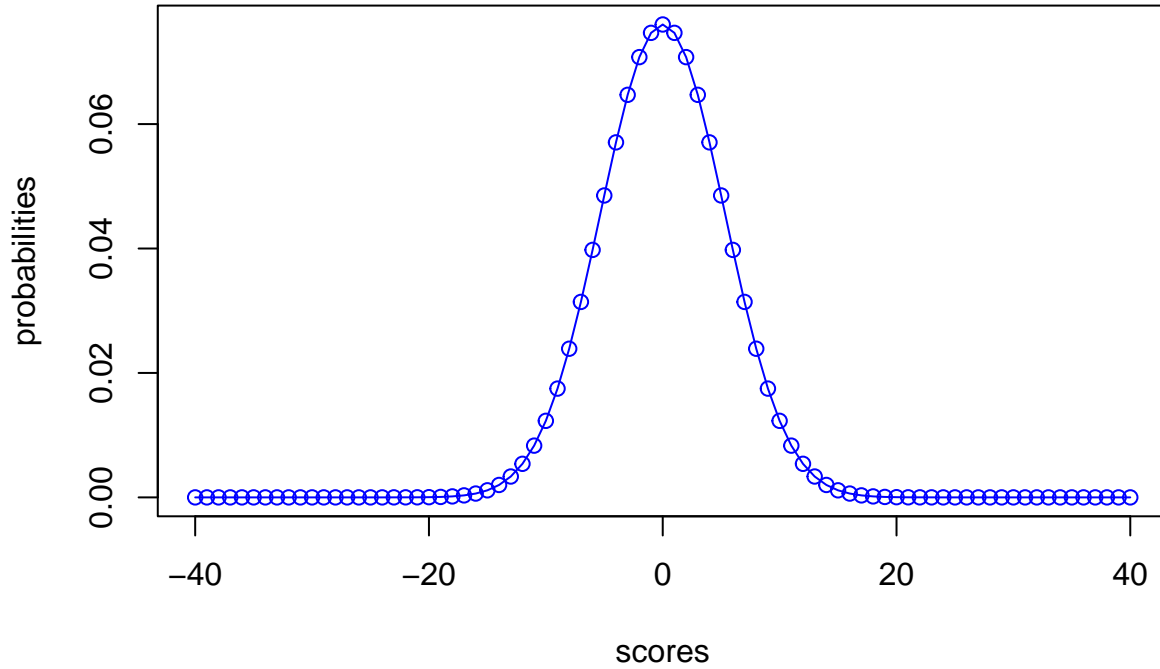
## Functionality

**QuaternaryProd** provides different functions for computing the probability of a score, probability mass function, p-value of a score and the domain of the Quaternary Dot Product Scoring Statistic. The probability mass function can be computed if given the margins of the table.

```
library(QuaternaryProd)

# Compute the probability mass function
pmf <- QP_Pmf(q_p = 20, q_m = 20, q_z = 20, q_r = 0, n_p = 20, n_m = 20, n_z = 20)

# Plot the mass function
plot(names(pmf), pmf, col="blue", xlab = "scores", ylab = "probabilities")
lines(names(pmf), pmf, col = "blue")
```



The package contains optimized functions for computing the p-value of a score. To compute the p-value of score we can use the following:

```
# Get the p-value of score 5
pval <- QP_Pvalue(score = 5, q_p = 20, q_m = 20, q_z = 20, q_r = 0,
                  n_p = 20, n_m = 20, n_z = 20)
pval

## [1] 0.1948157
```

```

# Compute the p-value only if it is statistically significant otherwise
# return -1
pval <- QP_SigPvalue(score = 5, q_p = 20, q_m = 20, q_z = 20, q_r = 0,
                    n_p = 20, n_m = 20, n_z = 20)
pval

## [1] -1

```

If the user is only interested in obtaining statistically significant p-values, then `QP_SigPvalue` is optimized for this purpose. In either case, the user is advised to compute the p-value of a score using the previous two functions which will be faster than computing the entire probability mass function and then computing the p-value. Finally, it is possible to also compute the probabilities of scores individually using `QP_Probability` and the support of the distribution using `QP_Support`. Since this package is written for the benefit of bioinformaticians, we will provide an example on how to apply this statistic to a publicly available network. One bioinformatic application is to test how well protein-protein causal networks can predict the regulators in gene expression data. In the last section of this Vignette, we present an example for computing this statistic over the STRINGdb network.

## Functionality for working with the Homo sapien causal network from STRINGdb

Here we provide functionality for using **QuaternaryProd** with the STRINGdb Homo Sapien causal network (version 10) provided under the [creative commons license](#).

### Compute Pvalues Over the Network

Given new gene expression data, we can compute the scores and p-values for all regulators in the STRINGdb Homo Sapien causal network using the specialized `RunCRE_HSAStrngDB` function. Note that the user can specify and use other causal networks with `RunCRE_HSAStrngDB`. We use the gene expression data sets that were used in [1]. The data sets contain the c-Myc and E2F3 expression signatures. Note that the results may differ from those reported in [2] since the network was parsed differently.

**Load Gene Expression Data** First, we load all the data sets. The gene expression data sets must have the following columns: 1- `entrez` column corresponding to the entrez id of the gene, 2- `pvalue` column corresponding to the pvalue of the gene, 3- `fc` column corresponding to the fold change of the gene. After we load the data sets, we make sure that there are no duplicated entrez ids in the data sets.

```

library(QuaternaryProd)

# Get gene expression data
e2f3 <- system.file("extdata", "e2f3_sig.txt",
                   package = "QuaternaryProd")
e2f3 <- read.table(e2f3, sep = "\t",
                  header = TRUE, stringsAsFactors = FALSE)
myc <- system.file("extdata", "myc_sig.txt",
                  package = "QuaternaryProd")
myc <- read.table(myc, sep = "\t",
                 header = TRUE, stringsAsFactors = FALSE)

# Rename column names appropriately
# and remove duplicated entrez ids in the gene expression data
names(e2f3) <- c("entrez", "pvalue", "fc")
e2f3 <- e2f3[!duplicated(e2f3$entrez),]

names(myc) <- c("entrez", "pvalue", "fc")
myc <- myc[!duplicated(myc$entrez),]

```

**Compute the Quaternary Dot Product Scoring Statistic over STRINGdb** We can now compute the Quaternary Dot Product Scoring Statistic over STRINGdb using the following:

```
# Compute the Quaternary Dot Product Scoring Statistic for only statistically
# significant regulators
```

```
quaternary_results <- RunCRE_HSAStrngDB(e2f3, method = "Quaternary",
                                       fc.thresh = log2(1.3), pval.thresh = 0.05,
                                       only.significant.pvalues = TRUE,
                                       significance.level = 0.05,
                                       epsilon = 1e-16, progressBar = FALSE,
                                       relations = NULL, entities = NULL)
```

```
## (1/5) Begin Processing...
## (2/5) 135 rows from gene_expression_data removed due
##       to entrez ids being unrepresented in network entities!
## (3/5) Loading STRINGdb causal network...
## (4/5) Computing P-values...
## (5/5) Done.
```

```
quaternary_results[1:4, c("uid", "symbol", "regulation", "pvalue")]
```

```
##           uid symbol regulation      pvalue
## 1 9606.ENSP00000345571  E2F1         up 6.160448e-08
## 2 9606.ENSP00000362592  RBBP4        down 3.095362e-05
## 3 9606.ENSP00000244741  CDKN1A        down 3.520453e-05
## 4 9606.ENSP00000274255   SKP2        down 7.772292e-05
```

RunCRE\_HSAStrngDB returns a data frame containing all the regulators of the String causal network. An efficient algorithm for computing the Quaternary Dot Product Scoring Statistic is presented in [2] and is implemented in **QuaternaryProd**. To further improve performance, RunCRE\_HSAStrngDB has an optional argument `only.significant.pvalues` which can be set to `TRUE` so that only the p-values of the statistically significant regulators are computed. P-values which are not statistically significant (i.e p-values greater than `significance.level`) are not reported, in which case the p-value is set to a value of -1. If the user wishes to compute p-values for regulators which are not statistically significant then the user should set the parameter `only.significant.pvalues = FALSE`. The regulators are ordered in increasing order of the p-values (Note: details on the columns of the data frame returned can be found in the help page for RunCRE\_HSAStrngDB). Finally, we see that this approach retrieves the signal regulator E2F1.

**Compute the Ternary Dot Product Scoring Statistic and the Enrichment test over STRINGdb** To compute the Ternary Dot Product Statistic over STRINGdb we can use the following:

```
ternary_results <- RunCRE_HSAStrngDB(myc, method = "Ternary",
                                     fc.thresh = log2(1.3), pval.thresh = 0.05,
                                     only.significant.pvalues = TRUE,
                                     significance.level = 0.05,
                                     epsilon = 1e-16, progressBar = FALSE,
                                     relations = NULL, entities = NULL)
```

```
## (1/5) Begin Processing...
## (2/5) 107 rows from gene_expression_data removed due
##       to entrez ids being unrepresented in network entities!
## (3/5) Loading STRINGdb causal network...
## (4/5) Computing P-values...
## (5/5) Done.
```

```
ternary_results[1:4, c("uid", "symbol", "regulation", "pvalue")]
```

```
##          uid      symbol regulation      pvalue
## 1 9606.ENSPO0000367207      MYC      up 3.453199e-06
## 2 9606.ENSPO0000351490      MAX      up 3.110565e-05
## 3 9606.ENSPO0000313199 No-Symbol      up 4.237870e-05
## 4 9606.ENSPO0000258962      SRSF1      up 2.875442e-04
```

We see that this method retrieves MYC as a significant regulator. To compute the Enrichment test over STRINGdb we can use the following:

```
enrichment_results <- RunCRE_HSAStringDB(myc, method = "Enrichment",
                                          fc.thresh = log2(1.3), pval.thresh = 0.05,
                                          only.significant.pvalues = TRUE,
                                          significance.level = 0.05,
                                          epsilon = 1e-16, progressBar = FALSE,
                                          relations = NULL, entities = NULL)
```

```
## (1/5) Begin Processing...
## (2/5) 107 rows from gene_expression_data removed due
##       to entrez ids being unrepresented in network entities!
## (3/5) Loading STRINGdb causal network...
## (4/5) Computing P-values...
## (5/5) Done.
```

```
enrichment_results[enrichment_results$symbol == "MYC", c("symbol", "regulation", "pvalue")]
```

```
##      symbol regulation      pvalue
## 39      MYC      up 0.001429373
## 40      MYC     down 0.001429373
```

We see that the Enrichment method also retrieves MYC as a significant regulator although not as significant as in the case of the Ternary Dot Product Scoring Statistic.

## References

- [1] Chindelevitch et al. (2012). Assessing statistical significance in causal graphs. BMC Bioinformatics, Volume 3, Issue 1, 2012, Page 35.
- [2] Carl Tony Fakhry, Parul Choudhary, Alex Gutteridge, Ben Sidders, Ping Chen, Daniel Ziemek, and Kourosh Zarringhalam. Interpreting transcriptional changes using causal graphs: new methods and their practical utility on public networks. BMC Bioinformatics, 17:318, 2016. ISSN 1471-2105. doi: 10.1186/s12859-016-1181-8.
- [3] Franceschini, A (2013). STRING v9.1: protein-protein interaction networks, with increased coverage and integration. In: 'Nucleic Acids Res. 2013 Jan;41(Database issue):D808-15. doi: 10.1093/nar/gks1094. Epub 2012 Nov 29'.